

Answer all the following questions **CALCULATORS ARE NOT ALLOWED.**

1. a) If  $2y + \sin(xy) = -1 + \tan x$ , find the equation of the tangent line to the graph of  $y$  at  $x = 0$ . (4 points)

b) Let

$$f(x) = \begin{cases} \frac{\sin(x-1)}{x^2 - 3x + 2}, & \text{if } x \neq 1 \\ A, & \text{if } x = 1 \end{cases}$$

Find the constant  $A$  if  $f$  is continuous at  $x = 1$ . (3 points)

2. Find the following integrals:

a)  $\int_0^{\pi/2} \frac{\sin x}{(1 + \cos x)^2} dx$  (4 points)      b)  $\int x(x^4 + 2x^2 + 1)^{20} dx$  (3 points)

3. a) Find an antiderivative of  $x\sqrt{1+x}$ . (4 points)

b) Show that  $g(x) = x^3 + ax + 2 = 0$ ,  $a > 0$  has exactly one real root. (3 points)

4. Find the following limits whenever they exist.

a)  $\lim_{x \rightarrow +\infty} x \sin \frac{1}{x}$  (4 points)      b)  $\lim_{x \rightarrow 0} x \sqrt{1 + \frac{1}{x^2}}$  (3 points)

5. Let  $f(x) = \frac{x^2 + 2x - 4}{x^2}$ . (10 points)

- Find the vertical and horizontal asymptotes of  $f$ , if any.
- Find the intervals where  $f$  is increasing and where it is decreasing.
- Find the relative extrema of  $f$ , if any.
- Study the concavity of  $f$  and find the point(s) of inflection(s).
- Sketch the graph of  $f$ .

6. An inverted conical tank has height 10 ft. and a circular top of radius 4 ft. (see figure 1). water is taken from the tank at a rate of 5 ft<sup>3</sup>/min. How fast the water level is decreasing when the height of the water is 6 ft. (5 points)

7. Find the maximum area of the rectangle inscribed in the curve  $x^2 + 4y^2 = 4$  and having sides parallel to the coordinate axes (see figure 2). (6 points)

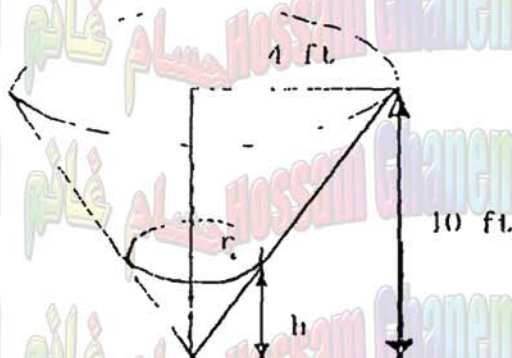


Figure 1

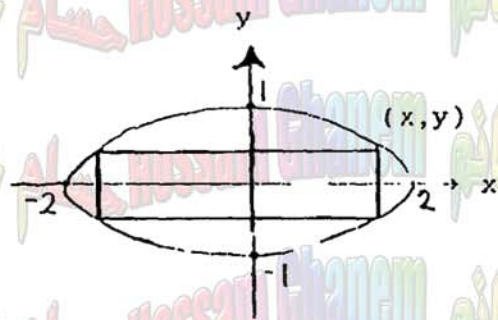


Figure 2.